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DISPERSION PHENOMENA IN A BOILING BED

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Wave phenomena in a one-dimensional boiling bed are considered. A dispersion equation is derived which shows that instability in a boiling bed is weak in a fairly great number of cases. The Korteweg–de Vries–Burgers equation is obtained for waves of small but finite amplitude in the bed. Oscillations at the fronts of gas bubbles in a boiling bed are investigated. The linear increase of density fluctuations with distance from the bed bottom and the jump of fluctuation at the upper boundary are explained.

The mathematical analysis of stability of equations of a boiling bed appeared in several publications (see, e.g., [1 – 3]) in which it is shown that a strong instability exponentially increasing with time occurs in such beds. However no allowance was made in these for the boundedness of the bed in space, and the existence of homogeneous boiling beds at low fluidization rates is not explained.

Here the analysis of dispersion and the investigation of wave phenomena in a boiling bed is based on expansion in a small parameter introduced in [4].

The simple model of the boiling bed described in [1] is used for deriving the dispersion equation. The pseudo-gas viscosity and the pressure of pseudo-gas in particles are neglected, and the viscosity of the fluidizing gas is taken into account only in the interaction force between particles and gas. The model is one-dimensional, i. e. all functions depend only on the vertical component x.

In this case the input equations are of the form

$$\rho_s \varepsilon \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x}\right) = -\varepsilon \frac{\partial p}{\partial x} - \rho_s \varepsilon G + \Phi$$

$$\rho_f (1-\varepsilon)\left(\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x}\right) = -(1-\varepsilon)\frac{\partial p}{\partial x} - \rho_f (1-\varepsilon) G - \Phi$$

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial \varepsilon u}{\partial x} = 0, \qquad \frac{\partial \varepsilon}{\partial t} - \frac{\partial}{\partial x}[(1-\varepsilon) v] = 0$$

$$\Phi = \frac{9}{2}\,\frac{\nu}{a^2}\,\rho_f\,\frac{\varepsilon H_0}{1-\varepsilon}\,(v-u)$$

where ρ_s and ρ_f are the densities of particles and gas, respectively; ε is the effective volume occupied by particles; u and v are the velocities of particle and gas, respectively; p is the pressure; G is the acceleration of gravity; Φ is the model force of phase interaction; a is the particle radius; ν is the kinematic viscosity of gas, and H_0.

is a dimensionless constant of the charge. Viscous terms are disregarded (except the inner-phase friction Φ), owing to the comparatively low viscosity of the fluidizing gas and of the particle pseudo-gas.

We introduce the dimensionless quantities t', u', v', Π', g' and α defined by

$$t = \frac{L}{V} t', \quad u = Vu', \quad v = Vv', \quad p = \rho_f V^2 \Pi', \quad G = \frac{V^2}{L} g', \quad \frac{\rho_f}{\rho_s} = \alpha$$

where L is a linear dimension of the equipment, V is the gas velocity ahead of the screen, and ρ_f and ρ_s are the densities of gas and particles, respectivelt. Unless otherwise stated, dimensionless quantities are used below and primes are omitted.

We pass to dimensionless variables, eliminate the pressure Π, and obtain for functions ε, u and v the system of three equations

$$\beta\left[\left(\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x}\right) - \alpha\left(\frac{\partial v}{\partial t} + v\,\frac{\partial v}{\partial x}\right)\right] = \frac{1}{(1-\varepsilon)^2}(v-u) - (1-\alpha)K \qquad (1)$$

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial(\varepsilon u)}{\partial x} = 0, \qquad \frac{\partial \varepsilon}{\partial t} - \frac{\partial}{\partial x}[(1-\varepsilon)v] = 0$$

$$K = \beta g, \quad \beta = R^{-1}$$

$$R = \frac{9\nu H_0 \alpha L}{2Va^2} = \frac{9\alpha H_0}{2}\,\frac{L}{a}\,\frac{1}{\mathrm{Re}_s}, \quad \alpha = \frac{\rho_f}{\rho_s}, \quad \mathrm{Re}_s = \frac{aV}{\nu}$$

The parameter R, defined in terms of Re_s – the Reynolds number – related to particles, was introduced in a three-dimensional problem in [4]. In many cases parameter R is considerable because La^{-1} exceeds α^{-1} by several orders of magnitude, while the Reynolds number Re_s is not very great in a real equipment. Subsequently we assume that $R \gg 1$.

To clarify the meaning of the term $(1-\alpha)K$ we analyze the solution of system (1) which corresponds to the stationary homogeneous mode

$$\varepsilon = \varepsilon_K = \mathrm{const}, \quad v = v_K = \mathrm{const}, \quad u = 0$$

Elementary computation shows that $v_K = [(1-\alpha)K]^{1/3}$ and $\varepsilon_K = 1 - v_K^{-1}$. In dimensional variables the formula for ε_K is of the form

$$\varepsilon_K = 1 - \left[\frac{\alpha}{1-\alpha}\,\frac{9\alpha H_0}{2a^2 G}\,V\right]^{1/3}$$

Note that ε_K is independent of the bed linear dimension. This is important for the subsequent analysis.

The dispersion equation for the linearized system (1) is of the form

$$\Omega = 3k(\varepsilon_K v_K) + i\beta(1-\varepsilon_K)^2[\alpha\varepsilon_K(\Omega + kv_K)^2 + (1-\varepsilon_K)\Omega^2] \qquad (2)$$

where Ω is the frequency and k is the wave number. Equation (2) defines two branches of function $\Omega = \Omega(k)$. The first of these is determined by expanding Ω in positive powers of the small parameter β

$$\begin{aligned}
&\Omega(k) = ck + i\beta\mu_1 k^2 - \beta^2\mu_2 k^3 + O(\beta^3), \quad c = 3\varepsilon_K v_K \\
&\mu_1 = (1-\varepsilon_K)^2\varepsilon_K v_K\{\alpha v_K(1-3\varepsilon_K)^2 + 9\varepsilon_K\}, \\
&\mu_2 = \mu_1(1-\varepsilon_K)^2\varepsilon_K\{3 + 3\alpha\varepsilon_K v_K - \alpha v_K\}
\end{aligned} \qquad (3)$$

It is easily verified that in this case $0 < \varepsilon_K < 1$, $(1 - v_K)\varepsilon_K = 1$, and $0 < \alpha < 1$. Both dispersion coefficients μ_1 and μ_2 are greater than zero. The second branch $\Omega = \Omega_d(k)$ is of the form

$$\Omega_\alpha(k) = -\frac{i}{\beta}a - [c + \Delta]k + O(\beta) \tag{4}$$

$$a = [(1-\varepsilon_K)^2(1-(1-\varepsilon_K)\varepsilon_K)]^{-1}, \quad \Delta = 2\alpha\varepsilon_K v_K[1-(1-\alpha)\varepsilon_K]^{-1}$$

i.e. Ω_d corresponds to waves rapidly attenuating with time and propagating against the fluidizing gas flow at the phase velocity $c + \Delta$; because of this, waves which obey the dispersion law (4) inside the bed are not considered here. However they must be taken into account in the vicinity of the bed upper boundary (see below).

Since in dimensionless variables the depth of bed and the bed phase velocity $c = 3\varepsilon_K v_K$ (for not excessively fluidized beds) are of the order of unity, the group velocity c_g of a wave packet and, consequently, the time of existence of a perturbation in the bed are also of the order of unity. The dispersion law (4) implies that a perturbation during its existence in the bed increases linearly with time as $1 + \beta\mu_1 k^2 t$, i.e. the instability is weak. It should be borne in mind that the wave number k is bounded, since for the considered model of the boiling bed only wave lengths not shorter than the scale of averaging used in the derivation of input equations have any meaning.

The law of dispersion implies that the amplitude of perturbations localized in space – wave packets – increases linearly with their progress through the bed. This accords with the linear increase of small density fluctuations along the bed height which is observed in experiments.

Waves moving from the depth of the bed are reflected by the upper boundary and are attenuated in accordance with the dispersion law (4). Thus in the narrow region close to the upper boundary perturbations are the result of two wave systems: waves arriving from the depth of the bed and those reflected from the upper boundary: this may explain the characteristic burst of fluctuations near the upper boundary [5]. The preceding analysis also indicates that the failry narrow region near the upper boundary does not affect waves of moderate amplitude inside the bed.

The considered model provides a simple description of waves of small but finite amplitude. It follows from (1) that

$$u = v - v_K^3(1-\varepsilon)^2 - \beta(1-\varepsilon)^2\left[\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) - \alpha\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right)\right] \tag{5}$$

Using the method of successive approximations, we obtain for u an expression in terms of ε and v that is accurate to within β^2 ; it corresponds to the dispersion branch (3), i.e. to waves propagating upward in the bed without attenuation. The equations of conservation of mass of particles and gas together with Eq. (5) imply that

$$\varepsilon_t + v\varepsilon_x + v_x\varepsilon - f_\varepsilon'\varepsilon_x = D \tag{6}$$

$$\varepsilon_t + v\varepsilon_x - (1-\varepsilon)v_x = 0 \tag{7}$$

where D is the sum of terms of order β and β^2 which take into account dispersion and dissociation. If D is disregarded, we have a model of a boiling bed without dispersion and dissociation. Such model was considered earlier in [4], where solutions with conservative discontinuities represent simple waves. It is reasonable to assume that the effect of dispersion and dissociation in the case of waves of finite but small amplitudes is small.

In analyzing solutions of quasi-simple waves [6] this can be taken into account by setting $v = F(\varepsilon) + \psi(x, t)$, where $F(\varepsilon)$ is the solution of the kind of a simple wave in a system without dissipation and dispersion, i.e. in (6) $D = 0$ and $\psi(x, t)$ is a correction of the order of the dispersion terms. Waves propagating inside the bed in a direction opposite to the flow of gas are neglected owing to their rapid attenuation.

We assume that $\varepsilon = \varepsilon_K + \varepsilon'$, where ε' is small, and linearize terms of order β and β^2.

By linearizing Eqs. (6) and (7) we rapidly obtain $\psi_x = D$, hence

$$\varepsilon_t' + [v - (1 - \varepsilon) f_\varepsilon']\varepsilon_x' = (1 - \varepsilon_K)D$$

With the use of iterations we reduce all derivatives of ε with respect to t in D to derivatives with respect to x, from which follows

$$\varepsilon_t' + [v - (1 - \varepsilon) f_\varepsilon']\varepsilon_x' = -\beta\mu_1\varepsilon_{xx}' - \beta^2\mu_2\varepsilon_{xxx}'$$

Taking into consideration that in a model without dispersion [4] $v = f(\varepsilon) + 1$ and $f(\varepsilon) = v_K^3\varepsilon(1 - \varepsilon)^2$, and rejecting terms of order $\beta\varepsilon_x'$, we obtain

$$\varepsilon_t' + h(\varepsilon)\varepsilon_x' = -\beta\mu_1\varepsilon_{xx}' - \beta^2\mu_2\varepsilon_{xxx}' \tag{8}$$

$$h(\varepsilon) = -v_K^3(1 - \varepsilon)^2(1 - 4\varepsilon) + 1$$

Function $h(\varepsilon)$ monotonically increases from $\varepsilon = 0$ to $\varepsilon = 1/2$ and monotonically decreases from $\varepsilon = 1/2$ to $\varepsilon = 1$; $h(\varepsilon_K) = 3\varepsilon_K v_K$ is the speed of sound in the bed with perturbations propagating in the direction of gas flow (3). Subsequently we consider only deviations from the stationary state $\varepsilon = \varepsilon_K$ of the bed, and write (8) in the form of the Korteweg – de Vries – Burgers equation

$$\varepsilon_t + h(\varepsilon_K + \varepsilon)\varepsilon_x + \beta\mu_1\varepsilon_{xx} + \beta^2\mu_2\varepsilon_{xxx} = 0 \tag{9}$$

It can be assumed that a perturbation originating at the screen becomes stabilized and is transformed into some stationary wave motion defined by Eq. (9) inside the bed. According to [6, 7] the solution of Eq. (9) is sought in the form $\varepsilon = \varepsilon(x - Wt)$, where W is the wave propagation velocity, $W - c = \delta > 0$, and δ is small; $c = 3\varepsilon_K v_K$ is the speed of sound in the bed with perturbations propagating in the direction of the fluidizing gas flow. For $x \to 0$ we have $\varepsilon = \varepsilon' = \varepsilon'' = 0$, i.e. the perturbation has already detached itself from the screen. Below, the screen coordinate is $x = 0$ and the right-hand semiaxis x corresponds to the bed interior. It can be readily shown that ε satisfies the equation

$$H(\varepsilon_K + \varepsilon) - W\varepsilon + \beta\mu_1\varepsilon' + \beta^2\mu_2\varepsilon'' = 0, \quad H(\varepsilon) = \varepsilon - (1 - \varepsilon)f \tag{10}$$

Singular points of Eq. (10) are determined by the condition

$$-W\varepsilon + H(\varepsilon_K + \varepsilon) = 0 \tag{11}$$

Equation (11) has a trivial solution $\varepsilon = 0$ which corresponds to the stationary state of the bed. Since waves of small amplitude are considered, only roots slightly different from $\varepsilon = 0$ are of interest. Note that for $\varepsilon = 0$ the tangent to the curve of function $H(\varepsilon_K + \varepsilon)$ is at an angle whose tangent is $h(\varepsilon_K) = c$, where c is the speed of sound. The roots of (11) correspond to the intersection of the straight line $y = W\varepsilon$ with the curve $y = H(\varepsilon_K + \varepsilon)$. Since $W - c = \delta$ is small, hence within the segment $0 \leqslant \varepsilon \leqslant 1 - \varepsilon_K$

there are, generally, only three solutions.

Three cases must be distinguished, viz: (1) $\varepsilon_K < 1/_2$, (2) $\varepsilon_K > 1/_2$, and (3) $\varepsilon_K \sim 1/_2$. Taking into account that in cases (1) and (2) we deal with states that are not much different from stationary, only the smallest root need to be taken into consideration and, consequently, it is possible to linearize function $h(\varepsilon_K + \varepsilon)$. Case (3) is not considered here because of its indefiniteness, which appears to be the consequence of some defect of the model.

Equation (11) now assumes the form $(c - W)\varepsilon + 1/_2 h_\varepsilon'(\varepsilon_K)\varepsilon^2 = 0$ whose singular points are $\varepsilon = 0$ and

$$\varepsilon_* = \frac{2\delta}{h_\varepsilon'(\varepsilon_K)} = \frac{(1-\varepsilon_K)^2}{1-2\varepsilon_K}\,\frac{\delta}{3} \tag{12}$$

The singular points are of the following kinds: (1) for $\varepsilon = 0$ we have a saddle point; (2) for $\varepsilon = \varepsilon_*$ we have either (a) a stable node when $\mu_1 < \mu_c$ or (b) a stable focal point when $\mu_1 > \mu_c$, where $\mu_c = 2(\nu\delta)^{1/2}$. For $\varepsilon_K < 1/_2$ we have $\varepsilon_* > 0$, and for $\varepsilon_K > 1/_2$, $\varepsilon_* < 0$. The considered solution corresponds to the separatrix emanating in the phase plane from the coordinate origin (a saddle point). The related equation for $\mu_1 > \mu_c$ corresponds to a packet of attenuating (slack) "solitons" in the bed [6]. Parameter ε_* corresponds to the mean total amplitude in the packet; it is subsequently called the amplitude of the packet. Now (12) defines the relation between the packet amplitude and its propagation velocity. If in the absence of packets $\mu_c < \mu_1$, then ε_* plays the part of amplitude and (11) defines the relation between the propagation velocity and the amplitude. The space scale of the wave process defined by the quasi-stationary solution of the described kind can only be determined numerically. It can be assumed that this scale is not great, since the dissipative term in Eq. (9) is of the order β, and the dispersion term of order β^2.

The considered here quasi-stationary solutions can be used for determining the pattern of strong discontinuities – shock waves – in a boiling bed. The solution of Eq. (10) leading from one point of equilibrium to another, i.e. a separatrix, is to be sought in such cases [7]. For $\varepsilon_K < 1/_2$ it is possible to join only compression discontinuities (upstream of the discontinuity ε is smaller than downstream of it), and for $\varepsilon_K > 1/_2$ only rarefaction discontinuities. For $\mu_c > \mu_1$ the discontinuity is oscillatory. A one-dimensional bubble or piston may be considered as a pair of discontinuities with the density of the bed between them lower than outside. It follows from the above that when such piston moves through the bed with the mean value of $\varepsilon_K < 1/_2$ and $\mu_c > \mu_1$, the forward boundary is oscillating and the rear one is clear-cut (it does not join), while for $\varepsilon_K > 1/_2$ and $\mu_c > \mu_1$ the rear boundary is oscillating.

Clear definitions and blurrings at the forward boundary of bubbles in a boiling bed were often observed experimentally [8]. Several papers dealt theoretically with this subject (see, e.g., [2, 3]). There are no publications in which oscillations at bubble boundaries are considered, although some experiments indicate their presence (see, e.g., the photographs in [8]).

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DYNAMIC EDGE EFFECTS IN ORTHOTROPIC ELASTIC SHELLS

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A development of the theory of the dynamic edge effect [1, 2] applied to the free vibrations of thin elastic orthotropic shells is given. It is assumed that the lines of curvature, the principal directions of elasticity and the lines along which the boundary conditions are given all coincide. The properties of the characteristic roots of the equations governing the kind of damping rate of the edge effects are investigated. Necessary and sufficient conditions are established for nondegeneration of the edge effects as a function of relationships between the principal curvatures and the shell elasticity coefficients.

1. Let us use the asymptotic method [1, 2] to evaluate the free vibrations frequencies of a thin elastic orthotropic shell whose principal directions of the material elasticity coincide with the coordinate lines x_1, x_2, which are the lines of principal curvatures. We assume compliance with the conditions for applicability of equations of Vlasov type and let us write the dimensionless equations for the preeminently bending vibrations modes as [3]

$$\nabla^4{}_d v + \frac{\partial^2\psi}{\partial\xi_1^2} + \chi\,\frac{\partial^2\psi}{\partial\xi_2^2} - \omega^2 v = 0 \tag{1.1}$$

$$\nabla_b{}^4\psi - \frac{\partial^2 v}{\partial\xi_1^2} - \chi\,\frac{\partial^2\psi}{\partial\xi_2^2} = 0$$

$$\nabla_d{}^4 = d_{11}\,\frac{\partial^4}{\partial\xi_1^4} + 2d_{12}\,\frac{\partial^4}{\partial\xi_1^2\partial\xi_2^2} + d_{11}^{-1}\,\frac{\partial^4}{\partial\xi_2^4}$$